Bachelor of Science (B.Sc.) Semester-III (C.B.S.) Examination

MATHEMATICS

(Advanced Calculus, Sequence & Series)

Paper—I

Time—Three Hours] [Maximum Marks—60

Note :— (1) Solve all the **FIVE** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (A) By using Lagrange's mean value theorem show that :

$$\frac{x}{1+x} < \log(1+x) < x, x > 0.$$ Hence show that

$0 < [\log (1 + x)]^{-1} - x^{-1} < 1, \forall x > 0.$ 6

(B) Let $F(x, y) = \dfrac{x^2y^2}{x^2y^2 + (x-y)^2}$. Then discuss the existence of iterated and double limits at (0, 0). 6

OR

(C) Let f(x, y) and g(x, y) be defined on the open region $D \subset R^2$. If f(x, y) and g(x, y) both are continuous at $\rho_o(x_o, y_o)$, then prove that $f(x, y)/g(x, y)$, $g(x_o, y_o) \neq 0$ is also continuous at $\rho_o(x_o, y_o)$. 6

(D) Expand sinxy in powers of $(x-1)$ and $(y-\pi/2)$ upto second degree terms by Taylor's theorem. 6

UNIT—II

2. (A) Find the envelope of the family of lines $x \cos\alpha + y \sin\alpha = l \sin\alpha \cos\alpha$, where the parameter is the angle α. Give the geometrical interpretation. 6

(B) Find the envelope of the straight line $\frac{x}{a}+\frac{y}{b}=1$ when $a^m b^n = c^{m+n}$, where c is a constant and the parameters are a and b. 6

OR

(C) Discuss the maximum and minimum values of $x^4 + 2x^2y - x^2 + 3y^2$. 6

(D) Determine the minimum value of $x^2 + y^2 + z^2$ subject to the condition $x + 2y - 4z = 5$ by using Lagrange's multiplier method. 6

UNIT—III

3. (A) Show that, the sequence whose n^{th} term is $\frac{3n+4}{2n+1}$, is bounded monotonic decreasing sequence for all $n \in N$ and tends to the limit 3/2. 6

(B) Let $<x_n>$ and $<y_n>$ be two sequences such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, where x and y are finite numbers, then prove that $\lim_{n\to\infty} (x_n - y_n) = x - y$. 6

OR

(C) Prove that if a sequence $< x_n >$ converges then it is a Cauchy sequence. 6

(D) Show that the sequence $< x_n >$ where $x_1 = 1$ and $x_n = \sqrt{(2 + x_{n-1})}$ converges to 2 by showing $< x_n >$ is monotonic and bounded, for all $n \in N$. 6

UNIT—IV

4. (A) Test the convergence of the series whose n^{th} term is $[(n^3 + 1)^{1/3} - n]$ by the comparision test. 6

(B) Test the convergence of the series :

$$2x + \frac{3}{8}x^2 + \frac{4}{27}x^3 + \ldots\ldots + \frac{(n+1)}{n^3}x^n + \ldots\ldots$$

for $x < 1$, $x > 1$ and $x = 1$ by the ratio test. 6

OR

(C) Test the alternating series :

$$\frac{2}{1} - \frac{3}{2^2} + \frac{4}{3^2} - \frac{5}{4^2} + \ldots\ldots\ldots$$

for convergence and also test for absolute convergence. 6

(D) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ by integral test. 6

UNIT—V

5. (A) If in the Cauchy's mean value theorem, $f(x) = x^2$ and $F(x) = x$ defined on [a, b], show that c is the arithmetic mean between a and b. 1½

(B) Show that the function :

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & ; \text{when } (x, y) \neq (0, 0) \\ 0 & ; \text{when } (x, y) = (0, 0) \end{cases}$$

is not continuous at (0, 0). 1½

(C) Find the envelope of the curve :

$tx^3 + t^2y = a$, parameter being t. 1½

(D) For $u = x^2 + y^2 + z^2$ subject to conditions $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$, find the Lagrange's equations in Lagrange's multiplier method. 1½

(E) Show that the sequence $< n/n+1 >$, $\forall\, n \in N$ is monotonic increasing and bounded. 1½

(F) If $< x_n >$ is a sequence in R, where

$$x_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots + \frac{1}{n}, \forall\, n \in N,$$

then evaluate $\lim\limits_{n \to \infty} |x_{n+1} - x_n|$ and show that it is monotonic increasing. 1½

(G) Test the convergence of the series $\Sigma \dfrac{1}{(\log n)^n}$ by root test. 1½

(H) Show that the series $\Sigma (-1)^{n-1} \dfrac{1}{n^2}$ is absolutely convergent. 1½